

Consider the conic with rectangular equation $8x^2 - y^2 + 32x + 14y - 1 = 0$.

SCORE: ____ / 25 PTS

[a] Find the co-ordinates of the focus/foci.

$$8x^2 + 32x - y^2 + 14y = 1$$

$$8(x^2 + 4x) - (y^2 - 14y) = 1$$

$$8(x^2 + 4x + 4) - (y^2 - 14y + 49) = 1 + 32 - 49 \quad (3)$$

$$(2) \quad 8(x+2)^2 - (y-7)^2 = -16 \quad (1)$$

$$(3) \quad \frac{(y-7)^2}{16} - \frac{(x+2)^2}{2} = 1$$

$$c^2 = 16 + 2 = 18 \rightarrow c = 3\sqrt{2} \quad (3)$$

$$(-2, 7 \pm 3\sqrt{2})$$

(2) (2)

[b] If the equation corresponds to a circle, find its radius.

If the equation corresponds to a parabola, find its directrix.

If the equation corresponds to an ellipse, find the endpoints of its minor axis.

If the equation corresponds to a hyperbola, find the equations of its asymptotes.

$$y-7 = \pm 2\sqrt{2}(x+2) \quad (2)$$

(2) (3)

Consider the polar equation $r = \frac{40}{3 - 5 \cos \theta}$.

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[a] Find the equation of the directrix.

$$r = \frac{\frac{40}{3}}{1 - \frac{5}{3} \cos \theta} \quad (2)$$

$$e = \frac{5}{3} \quad (2)$$

$$ep = \frac{40}{3} \quad (1)$$

$$\frac{5}{3}p = \frac{40}{3} \rightarrow p = 8$$

$$\begin{matrix} (2) & (2) & (2) \\ \hline x = -8 \end{matrix}$$

[b] Find the **rectangular** coordinates of the endpoints of **all** latera recta. **Do NOT convert the equation to rectangular.**

θ	r	(x, y)
0	-20	$(-20, 0)$
$\pi/2$	$40/3$	$(0, 40/3)$
π	5	$(-5, 0)$
$3\pi/2$	$40/3$	$(0, -40/3)$

(4)

$$\text{CENTER} = \left(\frac{-20 + -5}{2}, 0 \right) = \left(\frac{-25}{2}, 0 \right) \quad (2)$$

$$\begin{aligned} \text{FOCI} &= (2 * \frac{-25}{2}, 0) \text{ AND } (0, 0) \\ &= (-25, 0), (0, 0) \end{aligned}$$

ENDS OF L.R.

$$= \left(\frac{-25}{2}, \pm \frac{40}{3} \right), \left(0, \pm \frac{40}{3} \right)$$

(2) (2) (2) (2)

Consider the polar equation $r = 3 - 4 \sin \theta$.

SCORE: ____ / 30 PTS

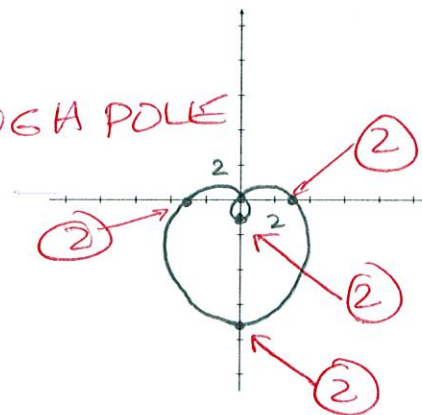
[a] Sketch the graph of the polar equation using the shortcut process shown in lecture / PowerPoint.

Label the scale on your axes clearly.

θ	r	(x, y)
0	3	(3, 0)
$\pi/2$	-1	(0, -1)
π	3	(-3, 0)
$3\pi/2$	7	(0, -7)

⑤ SHAPE

② GOES THROUGH A POLE



[b] Convert the equation to rectangular.

$$\begin{aligned} \textcircled{3} \quad r^2 &= 3r - 4r \sin \theta, \quad \textcircled{3} \\ x^2 + y^2 &= \textcircled{3} 3\sqrt{x^2 + y^2} - 4y, \quad \textcircled{3} \\ x^2 + y^2 + 4y &= 3\sqrt{x^2 + y^2} \\ (x^2 + y^2 + 4y)^2 &= 9x^2 + 9y^2, \quad \textcircled{3} \end{aligned}$$

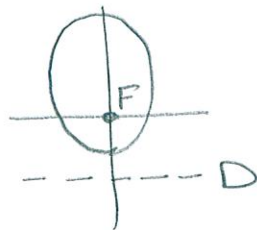
$$\begin{aligned} \text{OR} \quad r &= 3 - \frac{4y}{r}, \quad \textcircled{3} \\ r^2 &= 3r - 4y, \quad \textcircled{3} \\ \textcircled{3} \quad x^2 + y^2 &= 3\sqrt{x^2 + y^2} - 4y, \quad \textcircled{3} \\ x^2 + y^2 + 4y &= 3\sqrt{x^2 + y^2} \\ (x^2 + y^2 + 4y)^2 &= 9x^2 + 9y^2, \quad \textcircled{3} \end{aligned}$$

An ellipse has a focus at the pole and vertices with rectangular co-ordinates $(0, -4)$ and $(0, 7)$.

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- [a] Give polar co-ordinates for the vertices, using positive values of r and θ .

$$\left(4, \frac{3\pi}{2}\right), \left(7, \frac{\pi}{2}\right)$$



- [b] Find the polar equation of the ellipse.

$$r = \frac{ep}{1 - e \sin \theta}$$

$$4 = \frac{ep}{1 + e} \quad 7 = \frac{ep}{1 - e}$$

$$ep = 4 + 4e = 7 - 7e$$

$$11e = 3$$

$$e = \frac{3}{11}$$

$$\frac{3}{11}p = 4 + \frac{12}{11}$$

$$3p = 44 + 12$$

$$p = \frac{56}{3}$$

$$r = \frac{\frac{56}{11}}{1 - \frac{3}{11} \sin \theta}$$

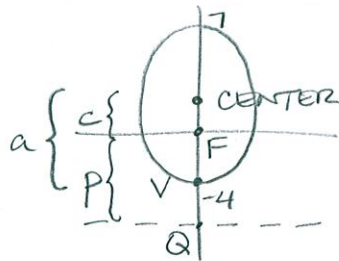
$$r = \frac{56}{11 - 3 \sin \theta}$$

(2) (2) SIMPLIFIED

[b]

Find the polar equation of the ellipse.

ALTERNATE SOLUTION



$$c = \frac{3}{2} \quad (2)$$

$$a = \frac{3}{2} - (-4) = \frac{11}{2} \quad (2)$$

$$e = \frac{c}{a} = \frac{3}{11} \quad (2)$$

$$e = \frac{VF}{VQ}$$

$$\frac{3}{11} = \frac{4}{p-4} \quad (1)$$

$$3p - 12 = 44$$

$$p = \frac{56}{3}$$

(1)

$$r = \frac{ep}{1 - e \sin \theta}$$

$$r = \frac{\frac{56}{11} \quad (1)}{1 - \frac{3}{11} \sin \theta \quad (1)}$$

$$r = \frac{56}{11 - 3 \sin \theta}$$

(2)

(2)

(2) SIMPLIFIED FINAL ANSWER

Name the shapes of the following graphs.

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[a] the graph with polar equation $r = \frac{13}{7 - \cos \theta}$

ELLIPSE

$2\frac{1}{2}$

[c] the graph with equation $7 + 7x^2 - 13y + 13y^2 = 0$

ELLIPSE

2

[e] the locus of points in the plane that are half as far from $x = 7$ as they are from $(7, 7)$

HYPERBOLA

3

[b] the graph with polar equation $r = 13 + 7 \sin \theta$

LIMACON WITH DIMPLE

$2\frac{1}{2}$

[d] the graph with polar equation $\theta = 5$

LINE

2

[f] the locus of points in the plane that are 7 units farther from $(7, -7)$ than they are from $(7, 7)$

HYPERBOLA

3

Consider the polar equation $r = 1 + \sin 2\theta$.

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The following symmetry tests do NOT indicate that the graph is symmetric:

$(-r, -\theta)$, $(-r, \pi - \theta)$ and $(r, \pi - \theta)$

$\theta = \pi/2$ POLAR AXIS $\theta = \pi/2$

- [a] Using the results above, along with the tests and shortcuts shown in lecture, determine if the graph is symmetric over the polar axis, $\theta = \frac{\pi}{2}$ and/or the pole. Summarize your conclusions in the table on the right.

NOTE: Run as FEW tests as needed to prove your conclusions are correct.

POLAR AXIS: $(r, -\theta)$ ② $r = 1 + \sin 2(-\theta)$

② $r = 1 - \sin 2\theta$

POLE: $(-r, \theta)$ ② $-r = 1 + \sin 2\theta$

② $r = -1 - \sin 2\theta$

$(r, \pi + \theta)$ ② $r = 1 + \sin 2(\pi + \theta)$

$r = 1 + \sin(2\pi + 2\theta)$

② $r = 1 + \sin 2\theta$

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

③

- [b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in [0, \pi]$ or $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ⑤

Find the rectangular equation of the parabola with focus $(-7, 5)$ and directrix $x = 13$.

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$$\text{VERTEX} = \left(\frac{-7+13}{2}, 5 \right) = \underline{(3, 5)} \text{ (3)}$$



$p = \text{DIRECTED DISTANCE FROM VERTEX TO FOCUS} = -7 - 3 = -10$

(4) $(y-5)^2 = 4(-10)(x-3)$

The number 4 is circled in red. Red arrows point from the circled 4 to the 4 in the equation. The 3 in the vertex (3, 5) is circled in red, and a red arrow points from it to the 3 in the equation. The -10 is circled in red, and a red arrow points from it to the -10 in the equation.

$$(y-5)^2 = -40(x-3)$$

The 1 in the coefficient of the y-term (y-5) is circled in red. The 1 in the coefficient of the x-term (x-3) is circled in red.